

Towards stochastic time-varying geological modeling ¹

Guillaume Caumon ²

November 17, 2010

Abstract

The modeling of subsurface geometry and properties is a key element to understand Earth processes and manage natural hazards and resources. We suggest this field should further evolve beyond pure data fitting approaches by integrating geological concepts to constrain interpretations or test their consistency. This process necessarily calls for adding the time dimension to 3D modeling both at the geological and human time scales. Also, instead of striving for one single “best” model, it is appropriate to generate several possible subsurface models in order to convey a quantitative sense of uncertainty. Depending on the modeling objective (e.g., quantification of natural resource, production forecast), this population of models can be ranked; inverse theory then provides a framework to validate (or rather invalidate) models which are not compatible with certain types of observations. We review recent methods to better achieve both stochastic and time-varying geomodeling and advocate that the application of inversion should rely not only on random field models but also on geological concepts and parameters.

KEYWORDS: geomodeling; uncertainty; structural restoration; inverse methods; geostatistics

¹Published in *Mathematical Geosciences* 42(5):555–569. DOI: 10.1007/s11004-010-9280-y.

²CRPG-CNRS, Nancy Universite, ENSG, rue du Doyen Marcel Roubault, 54500 Vandoeuvre-les-Nancy, France.

1 Introduction

The geological map has always been a central piece in geology, for it not only synthesizes observations, but also conveys many interpretations made from observations. Whereas a map provides a basis to answer a number of questions, it remains an incomplete representation, in particular when it comes to predicting the nature of rocks below the earth's surface.

During the last decades, geomodeling has been introduced to improve classical maps by providing an unambiguous 3D description of geological volumes. It has since then taken a prominent role in quantitative geosciences and management of natural resources; 3D structural modeling is now currently available in various software such as Dynamic Graphics (2009); Gemcom (2009); Intrepid Geophysics (2009); JOA (2009); Midland Valley (2009); Paradigm Geophysical (2009); Roxar (2009); Schlumberger (2009); Zaparo (2009). However, the “nice” pictures generated with such software should not hide that a model is at best an approximation of geological reality and may simply be wrong, especially away from observations. Indeed, geological data used to construct 3D models are generally not evenly distributed over the domain of study, have limited resolution, and provide an incomplete coverage of the physical quantities of interest (Bardossy and Fodor, 2001; Mann, 1993). Traditionally, geological concepts complement quantitative data to obtain a model deemed acceptable. However, modeling methods primarily put a strong focus on spatial data and only partially account for other geological observations and measurements (e.g., isotopic ages, mineral microstructures, strain analyses, etc). A need to better integrate the long and often complex geological history into modeling is therefore very acute to gain geologic insight and improve the quality of geomodels (Section 3).

A second avenue for investigations concerns the representation of uncertainties: not matter how good a 3D geological model may be, it will always be underconstrained to some extent. Characterizing this indeterminacy by appropriately varying model parameters is highly desirable, if only to evaluate risks when dealing with natural hazards and underground resources (Section 4).

Research towards such a stochastic and time-varying modeling would be extremely difficult to address with purely academic modeling systems. The complexity of geological objects and the nature of a computer-based modeling system indeed incur significant maintenance efforts. The building of an aca-

demio software *a nihilo*, although desirable by many, would take a significant energy just to reach the level of existing geomodeling packages. Currently, several commercial software vendors (Paradigm Geophysical, 2009; Roxar, 2009; Schlumberger, 2009) provide development frameworks to universities for them to capitalize on existing application programming interfaces and test new modeling concepts in an integrated manner while keeping the maintenance burden for themselves. Because good understanding of prior art is needed to address geomodeling challenges, Section 2 will now start by briefly reviewing existing methods to model 3D geological objects.

2 A short overview of geomodeling methods

2.1 Surface-based approaches

Since the early days of geomodeling, surfaces have gained a lot of interest to represent structural geological interfaces such as faults, horizons and unconformities (Mallet, 1988; Turner, 1992; Zanchi et al., 2009). One obvious reason for this choice is the ability of surface models to compactly represent complex shapes in 3D. From a mathematical standpoint, two main types of surface representations have been proposed:

Parametric surfaces use polynomial or rational equations describing the surface geometry using two parametric coordinates u and v . This representation is classically used in Computer Aided Design because it provides convenient user interaction. Moreover, it allows for data conditioning as described for instance by Piegl and Tiller (1997). Parametric surfaces have been used by several authors to represent 3D geological structures (de Kemp, 1999; de Kemp and Sprague, 2003; Fisher and Wales, 1992; Gjøystdal, Reinhardsen and Åstebøl, 1985; Hoffman, Neave and Klein, 2003; Sprague and de Kemp, 2005; Zhong et al., 2006). In these approaches, discontinuities are addressed using one parametric patch per connected component (fault segment or continuous horizon inside a fault block). These surfaces need then to be truncated along discontinuities for graphical display and structural model queries.

Polygonal surfaces consist of a network of nodes connected by polygons. Regular rectangular connection patterns are very easy to represent on a computer and are well suited to the creation of corner-point reservoir

grids (Fremming, 2002). However, they have shortcomings to represent complex structures. Triangular surfaces provide an interesting alternative. From a mathematical standpoint, they have the nice properties of simplicial meshes. From a geological standpoint, they can conform without degeneracy to all types of geometry and topology (densely faulted domains, complex intrusive or erosive contacts, etc.), and can be adaptively refined where needed (Caumon et al., 2009; Jessell, 2001; Lemon and Jones, 2003; Mallet, 1988, 2002).

Whatever the mathematical model retained, surfaces should honor representational validity rules (finite extension, orientability, non-intersection (Hoffmann, 1989)) and geological validity rules (non-intersection of rock boundaries, absence of dangling surface edges except for laterally dying and synsedimentary faults (Caumon, Sword and Mallet, 2004)).

2.2 Volume-based approaches

Isolated surfaces are the 3D counterpart of the spaghetti format in GIS: they do not necessarily bound well-defined volumes. The simplest volume model is a Cartesian grid whose blocks are flagged depending on the geological unit they belong to (Gjøystdal, Reinhardsen and Åstebøl, 1985). As shown on Figure 1(A), geological interfaces are then stair-stepped; increasing model resolution helps improving model accuracy, but incurs significant memory usage and increases the number of model parameters. Adaptive Cartesian grids such as octrees may be used to more compactly represent structures by using local refinement in areas of high geometric complexity. However, such Cartesian representations do not directly represent fault offset nor locally varying anisotropy related to rock deformations.

Stratigraphic (corner-point) grids (Fig. 1(B)) address both problems by conforming grid blocks to most structural interfaces. Construction is often achieved by extrusion of a 2D grid vertically (Johnson and Jones, 1988; Hoaglund and Pollard, 2005; Swanson, 1988) or along so-called pillars which define a direction field tangent to the main faults (Bennis et al., 1996; Chambers et al., 1999; Fremming, 2002; Mallet, 2002). Alternatively, grids may be warped to conform to structural interfaces (Hoffman, Neave and Klein, 2003; Lasseter and Jackson, 2004).

These grids have become standard in the modeling of underground reservoirs. Indeed, they approximate the coordinate transform to apply geosta-

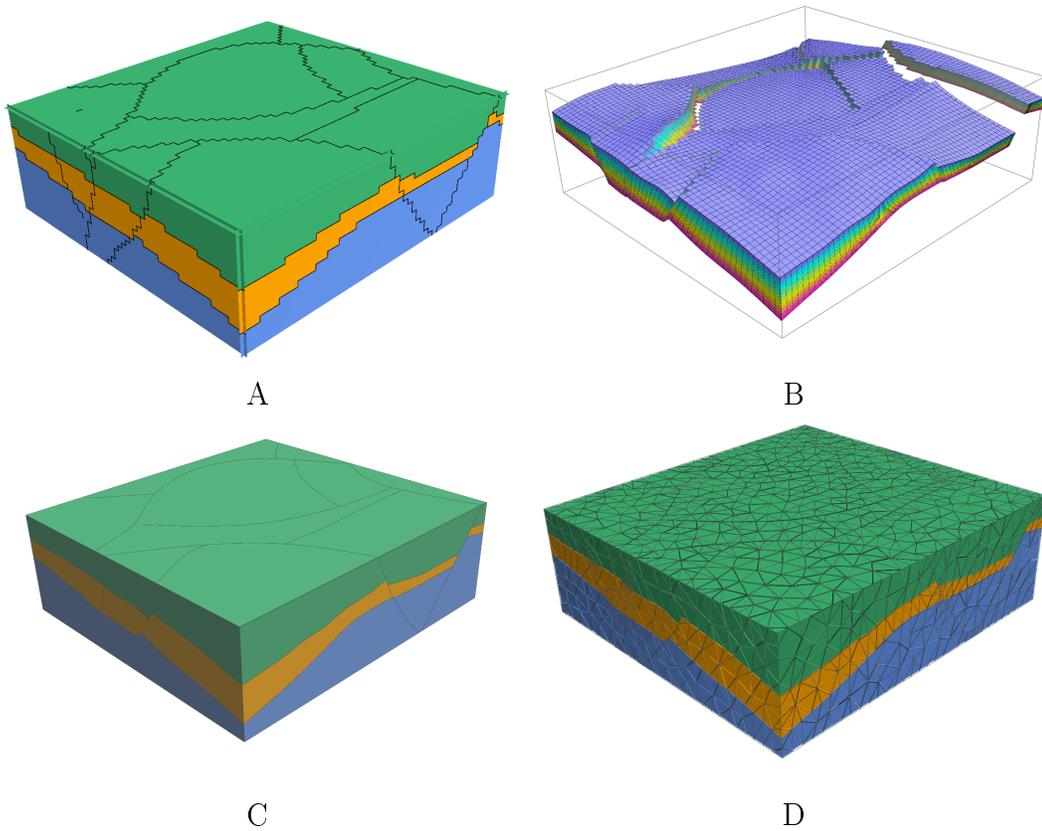


Figure 1: For a given structural model, several volumetric representations may be built, for instance a Cartesian grid (A), a corner-point stratigraphic grid (B), a boundary representation (C) or a tetrahedral mesh (D). Structural model courtesy of Total.

tistical methods in depositional space, thereby accounting for depositional heterogeneities (Mallet, 2004). Also, they offer a support to finite-volume based flow simulation. However, conforming to geological structures generally yields non-orthogonal grids, which may introduce distortions in geostatistical models (Gringarten et al., 2008; Jayr et al., 2009) and discretization errors in flow simulation (Wu and Parashkevov, 2009). Moreover, from a practical standpoint, these grids are very difficult to create in the presence of low-angle faults and subhorizontal fault contacts. This explains why Figure 1(B) only covers a subset of the volume of study. These shortcomings make corner-point grids difficult to apply at basin scale and to igneous and metamorphic formations.

Boundary representations, or sealed geological models, stitch surfaces together to define rock volumes (Fig. 1(C)). The full structural complexity can therefore be captured by these models, which have been widely used in geomodeling (Apel, 2006; Caumon, Sword and Mallet, 2004; Euler, Sword and Dulac, 1998; Frøyland et al., 1993; Gjøystdal, Reinhardsen and Åstebøl, 1985; Lemon and Jones, 2003; Mello and Henderson, 1997; Zhong et al., 2006). Analytical properties may be defined within each region to support seismic velocity modeling (Gjøystdal, Reinhardsen and Åstebøl, 1985; Guiziou, Mallet and Madriaga, 1996; Sword, 1991) or numerical integration with the boundary element method (Thomas, 1993; Maerten, Pollard and Karpuz, 2000). When a higher level of detail is needed, conformable meshes are generated within each region. Among these, tetrahedral meshes are simple and can in principle adapt to complex boundary geometries; as for triangulated surfaces, tetrahedral level of detail can be variable in space to honor data or geological features (Frank, Tertois and Mallet, 2007; Zhang and Thurber, 2005). In spite of recent advances in mesh generation (Tournois et al., 2009), accounting for complex constraints such as sharp geological contacts, thin layers or fracture networks is still a challenge (Mustapha and Mustapha, 2007; Owen, 1998).

One way to address these mesh generation problems is to use implicit surfaces or level sets to represent geological structures. In this representation, some structural interfaces are equipotential surfaces of some 3D scalar field $f(x, y, z)$. Although this idea is quite old (Houlding, 1994), it has democratized for the last few years only thanks to widely available computer memory. Several approaches have been described to build the 3D scalar field $f(x, y, z)$ from a set of data points, using radial basis functions (Cowan et al., 2003), dual kriging with polynomial drift (Calcagno et al., 2008; Chilès et al.,

2004), or discrete interpolation on Cartesian grids (Ledez, 2002) or tetrahedral meshes (Frank, Tertois and Mallet, 2007; Moyen et al., 2004). As compared to surface-based structural modeling, implicit methods are very attractive because they provide some built-in model consistency rules, and do not rely on data-to-surface projections which raise a number of problems in classical approaches.

2.3 Heterogeneity modeling

Notwithstanding the intrinsic value of 3D structural mapping, it is often only a step to generate petrophysical models which can be later used in numerical simulation of physical processes. This petrophysical modeling is the realm of Geostatistics (Chilès and Delfiner, 1999; Deutsch, 2002; Goovaerts, 1997; Journel and Huijbregts, 1978; Matheron, 1970; Rémy, Boucher and Wu, 2008). Shortly, these methods provide either an estimate of expected values at all locations or a set of simulated possible values which honor some random function model. A significant challenge in geostatistics is to define stationary domains where the intrinsic stationarity hypothesis is deemed acceptable, and to estimate spatial trends when sharp domain boundaries cannot be defined. Such trends may come from ancillary information provided by geophysical data, or from geological knowledge (Kedzierski et al., 2008; Massonnat, 1999).

3 The time dimension

Geological interpretation highly relies on one's understanding of a succession of geological events through time. However, most current geomodeling techniques only provide limited or indirect integration of this temporal dimension. Indeed, a model honoring basic topological validity rules is not necessarily compatible with a plausible geological history. The concepts of time-stratigraphy (Wheeler, 1958; Vail, 1987; Mallet, 2004) introduce geological time more directly into interpretation and geomodeling methods. From identified chronostratigraphic surfaces, it is possible to define a mapping between present-day physical space to chronostratigraphic space, where depositional heterogeneities can be modeled using geostatistical methods (Mallet, 2004; Caumon, Grosse and Mallet, 2004; Gringarten et al., 2008; Jayr et al.,

2009). In this view, chronostratigraphic horizons are seen as the equipotentials of the depositional time.

3D model quality controls reviewed by Caumon et al. (2009), such as layer thickness variations and fault juxtaposition diagrams, are also connected to depositional and kinematic concepts. Such principles may be included directly in the modeling method, for instance by constraining sedimentation and compaction rates with thickness constraints (Kaven, Mazzeo and Pollard, 2009; Mallet, 2002, p.269). Structural kinematic constraints may also be used during model construction to generate developable horizons (Thibert, Gratier and Morvan, 2005) or compatible geometry of fault surfaces (Thibault et al., 1996).

A more direct integration of time as an explicit model parameter is readily available in process-based geological simulations of sedimentary processes (Harbaugh et al., 1999) or tectonic processes (e.g., Braun, 1993; Jessell and Valenta, 1996). In principle, inverse theory may be used to constrain these models to spatial data (Cross and Lessenger, 1999). However, in practice, only sparse observations may be accounted for. Moreover, present observations result from a complex succession of depositional or setup processes, diagenetic and tectonic transformations; process-based models are generally concerned with only one of these aspects.

An alternative is therefore to start from 3D models built with current geomodeling methods, and perform balanced restoration to move back in time. Structural geologists have been performing such restorations for decades, mostly on cross-sections using geometric criteria (Dahlstrom, 1969; Elliott, 1983). Because cross-section restoration assumes planar strain, a significant effort has been made to adapt structural restoration techniques to 3D structural models during the last fifteen years. This first concerned geological horizons (Gratier, Guillier and Delorme, 1991; Rouby, Xiao and Suppe, 2000; Dunbar and Cook, 2003), then volumes, using either geometric methods (Mello and Henderson, 1997) or geomechanical principles (Muron, 2005; Maerten and Maerten, 2006; Moretti, 2008; Guzowski et al., 2009). As compared to other approaches, geomechanical restoration is able to reproduce heterogeneous mechanical behavior depending on rock type. Geomechanical restoration is applied on conformable meshes by specifying boundary conditions to sequentially flatten layer hanging walls. However, mesh generation itself is often time-consuming, and honoring fine-scale structural features tends to generate a very large number of elements. This limitation has been addressed by Durand-Riard, Caumon and Muron (2010): using an implicit

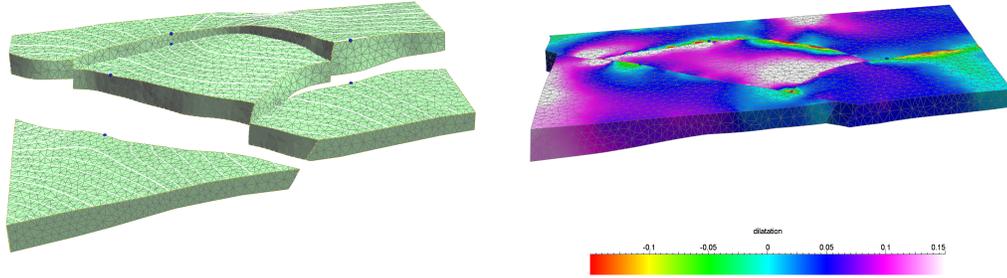


Figure 2: Present state (left) and restored state (right) of a faulted structural model. Dilation between restored and present states is color coded (model courtesy of Pierre Muron).

scalar field $f(x, y, z)$ to represent chronostratigraphic horizons, geomechanical restoration can be performed on a coarser mesh, conforming only to faults. In the presence of unconformities, several scalar fields may be used, which also allows for easily making hypotheses about the amount of eroded material.

Restoration can be used for three major purposes. First, it helps producing geometries which are kinematically acceptable from limited data. Second, it provides ways to assess the interplay between sedimentary and tectonic processes and to analyze basin evolution by providing a view at paleostructures. Last, it generates a 3D map of rock strain which can be used to predict fracture location, under the assumption that the deformation mechanism is correct (Fig. 2).

As compared to cross-section balancing, the quantitative usage of restoration for correcting 3D structural interpretations is still an active topic, which calls for fast model updating methods (Section 4). Other challenges lie in the definition of appropriate boundary conditions (Guzowski et al., 2009) and choice of a good rheological behavior to obtain realistic deformations. Coupling 3D restoration with quantitative sedimentological models is another area for future research.

At a shorter time scale, time-varying geological models mostly aim at representing fluid transport, reactive or not, and geomechanical transformations such as subsidence during hydrocarbon reservoir depletion, stability of rocks in underground cavities, etc. Many of these problems are currently ad-

dressed by simulating numerical processes on a static geomodel, and possibly assimilating observations to update the model (Section 4.2).

4 The uncertainty dimension(s)

Progresses in geomodel representation, construction and restoration can never remove ambiguity present in subsurface data (Bond et al., 2007). In many cases, creating several possible models reflecting the uncertainty is therefore preferable to building one single “best” model which tends to smooth features in areas of high local uncertainty. Indeed, this smoothing effect is often unrealistic, and may lead to significant biases in the modeling output (e.g., simulated behavior of an aquifer or hydrocarbon field). Multi-realization approaches better reproduce realistic geological features and assess the impact of subsurface uncertainty on modeling response, as demonstrated by many authors (Charles et al., 2001; Manzocchi et al., 2008).

Many geostatistical methods exist to sample petrophysical uncertainties using either classical two-point covariance, multi-point statistics carried by a training image or object models. However, the application of geostatistical simulation calls for defining stationary domains and providing global target statistics, whereas both are questionable. Therefore, realistic uncertainty assessment methods must go beyond standard geostatistical simulation. For example, uncertainty about a global parameter (e.g. global rock type proportion) can be assessed by spatial resampling from geostatistical realizations (Journel, 1993; Norris, Massonnat and Alabert, 1993). This method is a good way to assess uncertainty about a global parameter while honoring spatial variability models. However, to be representative, it should be performed on several realizations sampling the prior uncertainty distribution of this parameter (Caumon et al., 2004; Maharaja et al., 2008).

4.1 Structural model uncertainties

We will now review methods to look into uncertainty about geological structures. Indeed, structural uncertainty is often first-order as compared to other types of subsurface uncertainty, especially when observations are sparse. Even in the presence of 3D seismic data, poorly known velocity model and interpretation errors may induce significant uncertainties about position and relative layout of structural objects (Thore et al., 2002; Bond et al., 2007).

Perturbation or randomization of the structural model parameters are therefore needed to sample these uncertainties, then assess their impact on volumetrics, heterogeneities and response of hydrodynamic or other physical forward models.

With most structural parameterizations reviewed in Section 2, stochastic structural modeling is very difficult, or only samples a subset of the possibilities. For instance, Abrahamsen (1992); Charles et al. (2001); Goff (2000); Lecour et al. (2001) have proposed to perturb a reference structural model by moving surfaces along some pre-defined direction field, generally vertical or aligned along faults. The amount of surface displacement is evaluated by simulating 2D realizations of a 2D Gaussian random field or p-field simulation to sample from an existing uncertainty map. Another approach may use free-form deformation methods to interpolate a displacement field and distort discrete meshes (Caumon, Tertois and Zhang, 2007). However, this approach warps the whole model and does not easily allow for different displacement fields across faults. In hard rocks, Guillen et al. (2008) propose perturbing the geometry of a reference model using random sampling of region boundaries and mathematical morphology operators. Because all these methods can only generate models with the same topology, Hollund et al. (2002) and Holden et al. (2003) introduce a fault operator which can be introduced to alter fault block definitions and connectivity. Fault/horizon consistency is maintained by applying this operator on pillar-based corner-point grids. This practical decision is convenient to directly assess the effect of structural uncertainty on flow predictions in reservoirs, but can only simulate faults aligned on grid pillars. The range of structural uncertainty which can be represented with this approach is therefore smaller than desirable.

Constrained editing of sealed geological models is more flexible, but its implementation is very complicated, and difficult to automate without infringing model validity (Euler, Sword and Dulac, 1998; Caumon, Sword and Mallet, 2004). Recently, promising works have been presented to automatically update or perturb implicit models on tetrahedral meshes (Caumon, Tertois and Zhang, 2007; Tertois and Mallet, 2007; Cherpeau, Caumon and Lévy, 2009). In this case, perturbation of a given implicit model simply amounts to adding realizations of a random field $r(x, y, z)$ to the 3D scalar field $f(x, y, z)$. In the frame of estimation, Aug et al. (2004) has established a relationship between the local uncertainty of the 3D scalar field and the probability for the implicit surface to go through a point, under Gaussian assumption. In the frame of stochastic simulation, the approach to sample

uncertainties of Caumon, Tertois and Zhang (2007) is still heuristic, and only imposes values of the random field to be 0 at exact data location (in first approximation, the isosurface at location (x, y, z) is displaced by a distance equal to $r(x, y, z)/\|\nabla f(x, y, z)\|$). In practice, the random field $r(x, y, z)$ is controlled by a locally varying anisotropy aligned on the gradient ∇f of the scalar field f . Variogram range along this gradient controls layer thickness variations in the case of chronostratigraphic models; ranges orthogonal to ∇f control the sinuosity of the perturbation. Although additional work is needed to account for more complex types of uncertainties about geological surfaces (Thore et al., 2002), this method benefits from the built-in topological integrity of implicit models and does not make simplifying assumptions on structural features. When coupled with constructive solid geometry principles and random tree generation rules, it also allows for significant topological changes (Cherpeau, Caumon and Lévy, 2009).

4.2 Updating prior geological uncertainties

The methods we have reviewed so far are based on spatial data and geological concepts to build geomodels. One of the target applications of geomodels is to simulate physical processes (flow, heat transfer, wave propagation, etc.). The result of these numerical simulations may differ from the actual observations (flow rates / pressures at wells, heat flux, seismic waveform, etc.). Inverse theory is concerned with the search for model parameters compatible with this type of observation (Tarantola, 1987). Essentially, inverse methods look for the probability distribution $f_{post}(\mathbf{m}|\mathbf{d})$ of model parameters \mathbf{m} given a set of observations \mathbf{d} . Following Bayes' rule, this probability is proportional to the product of the probability of model parameters $f_{prior}(\mathbf{m})$ before making the observations \mathbf{d} times the likelihood probability $f(\mathbf{d}|\mathbf{m})$ of the data for a given model. This data likelihood probability is derived from the difference between the true and simulated data, generally under the Gaussian assumption of both measurement and forward simulation errors.

This theoretical framework is very general, but raises a number of challenges when applied to complex geomodels. Structural parameters are very diverse, and may include the absence or presence of a fault, the connection (or not) of two faults, the geometry of a layer, etc. It is not currently possible to cast such a heterogeneous set of features into a random vector of parameters characterized by covariances. Finding a model parameteriza-

tion which honors geological consistency is very difficult, and often relies on simplifications (Lelièvre, Oldenburg and Williams, 2008). In the case of random fields describing petrophysical heterogeneity, gradual deformation (Hu, Blanc and Noetinger, 2001) and probability perturbation method (Caers and Hoffman, 2006) provide effective ways of blending models while preserving their consistency. Ensemble Kalman Filter offers another avenue for updating stochastic models through data assimilation, generally under Gaussian assumption (Evensen, 2007; Aanonsen et al., 2009). Notably, this approach benefits from the time-varying nature of most inverse problems by successively updating the ensemble of models at successive time steps. However, the application of these methods to structures can account for continuous uncertain parameters, but not categorical parameters or different structure scenarios. Such simplifications may prevent many possible configurations to ever occur in a realization; this is not appropriate for sound uncertainty assessment.

To avoid simplifying assumptions, it can be convenient to consider that an ensemble of possible models define the prior uncertainty. Solving the inverse problem then amounts to eliminating those of the prior models which are not consistent with observations. This may be achieved naively by simulating the forward physical model for every single model in the prior space. This is not practically feasible due to the high computational cost of forward computations. To address this problem, Suzuki and Caers (2008); Suzuki, Caumon and Caers (2008) propose computing a distance between prior models (Fig. 3(A)), and then use this distance to search more efficiently for acceptable models (Fig. 3(C)). In the case of history matching, the Hausdorff distance between structural models was shown to be correlated to flow response (Fig. 3(C)). Discrete optimization techniques (Sambridge, 1999a,b; Brin, 1995) can then exploit this to find rapidly models which are compatible with observed production history. Scheidt and Caers (2009) also propose to use distances to project models into a feature space in order to speed selection of representative models. A practical problem with these approaches is that the prior set of models must be very large to cover the space of uncertainty. Therefore, further research is still needed to generate new and consistent models in a given area of the search space, as with gradual deformation or probability perturbation methods.

Another avenue for further research is to look into ways of computing the likelihood of a structural model using concepts related to geological time (Section 3). For instance, characterization of rock microstructures provides

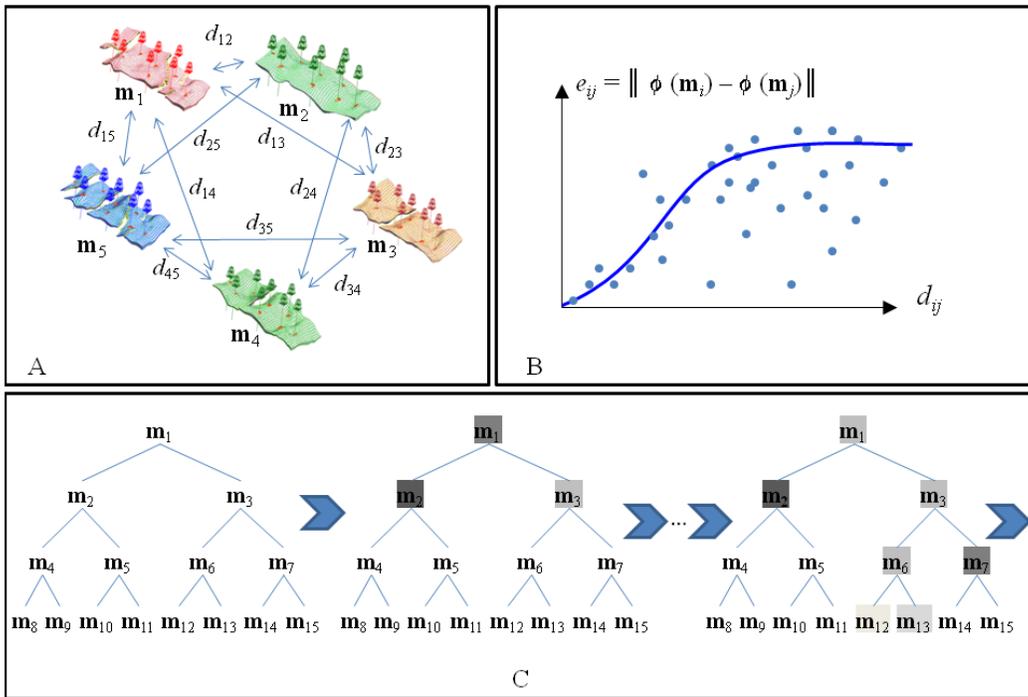


Figure 3: Sampling from a population of models to solve a history matching problem (Suzuki, Caumon and Caers, 2008). Hausdorff distances between models are computed (A), and are correlated differences in flow response (B). Efficient search of model space (C) can then be achieved with a proximity tree (Brin, 1995). The error magnitude between simulated and actual production is shown as grey intensity.

clues about the successive phases of deformation of a rock. Comparing such micro-scale observations to retrodeformations obtained by 3D restoration could provide indices about 3D structural model consistency.

5 Conclusions

Ideally, geological models should result from numerical simulations of the physical processes which led to the present state (Tarantola, 2006). The parameters of these processes should be inferred from qualitative observations, and matched with all quantitative data using stochastic inverse methods. The ensemble of models obtained that way would describe our level of knowledge about a geological domain; every new observation would then reduce the number of possible models. Currently, this vision cannot be put into practice (and this will probably remain so for a while): physical processes are coupled and still being described and improved by the geosciences community; forward model parameters are unlikely to be multivariate Gaussian; time-varying boundary conditions are poorly constrained; problem dimensionality is huge, even with parsimonious models.

I advocate a more pragmatic approach towards the integration of time and uncertainties in 3D geological modeling. Advances in the specifications and formulations of 3D model representations opens new perspectives to more easily generate possible structural models honoring prior geological concepts. For each of these models, a vast range of methods can assess petrophysical uncertainty. In the future, we believe that further progresses in parameterization of geological models will help addressing inverse problems to reconcile numerical simulation of physical processes and observations. Geological time may be incorporated in this parameterization through indirect rules or backward process-based simulation. Additional reasons to be optimistic are to be found in sensitivity analysis (to find the most significant parameters to a given problem), progresses in optimization methods, and hardware evolution. All these will hopefully converge towards making truly integrated, stochastic and time-varying models a reality.

Acknowledgements

This work benefited from a number of influences and contributions from colleagues and students and from the financial support of the Gocad research Consortium (www.gocad.org). This is CRPG-CNRS contribution 2049.

References

- Aanonsen SI, Nøevdal G, Oliver DS, Reynolds AC, Vallès B (2009) The ensemble kalman filter in reservoir engineering – a review. *SPE J* 14(3):393–412 13
- Abrahamsen P (1992) Bayesian kriging for seismic depth conversion of a multi-layer reservoir. In: Soares A (ed.) *Geostatistics Troia '92* vol 1 385–398 Kluwer, Dordrecht 11
- Apel M (2006) From 3d geomodelling systems towards 3d geoscience information systems: Data model, query functionality, and data management. *Comp Geosci* 32(2):222–229 6
- Aug C, Chiles JP, Courrioux G, Lajaunie C (2004) 3D geological modeling and uncertainty: the potential field method. In: Leuangthong O, Deutsch CV (eds.) *Geostatistics Banff, Proc seventh International Geostatistics Congress* 145–154 Kluwer, Dordrecht 11
- Bardossy G, Fodor J (2001) Traditional and new ways to handle uncertainty in geology. *Nat Resour Res* 10(3):179–187 2
- Bennis C, Sassi W, Faure JL, Hage Chehade F (1996) One more step in gocad stratigraphic grid generation: Taking into account faults and pinchouts. In: *Proc European 3D Reservoir Modelling Conference (SPE 35526)* 4
- Bond C, Gibbs A, Shipton Z, Jones S, Stracher G (2007) What do you think this is? Conceptual uncertainty in geoscience interpretation. *GSA TODAY* 17(11):4–10 10
- Braun J (1993) Three-dimensional numerical modeling of compressional orogenies: Thrust geometry and oblique convergence. *Geol* 21(2):153 8
- Brin S (1995) Near neighbor search in large metric spaces. In: *Proc. International Conference on Very Large Data Bases* 574–584, Morgan Kaufmann, San Francisco 13, 14

- Caers J, Hoffman T (2006) The probability perturbation method: a new look at bayesian inverse modeling. *Math Geol* 38(1):81–100 13
- Calcagno P, Chilès JP, Courrioux G, Guillen A (2008) Geological modelling from field data and geological knowledge Part I. Modelling method coupling 3D potential-field interpolation and geological rules. *Phys Earth Planet Inter* 171(1-4):147–157 6
- Caumon G, Collon-Drouaillet P, le Carlier de Veslud C, Sausse J, Viseur S (2009) Teacher’s aide: 3D modeling of geological structures. *Math Geosci* 41(9):927–945 4, 8
- Caumon G, Grosse O, Mallet JL (2004) High resolution geostatistics on coarse unstructured flow grids. In: Leuangthong O, Deutsch CV (eds.) *Geostatistics Banff, Proc. seventh International Geostatistics Congress* 703–712 Kluwer, Dordrecht 7
- Caumon G, Strebelle S, Caers JK, Journel AG (2004) Assessment of global uncertainty for early appraisal of hydrocarbon fields. In: *Proc SPE Annual Technical Conference and Exhibition (SPE 89943)* 10
- Caumon G, Sword CH, Mallet JL (2004) Building and editing a Sealed Geological Model. *Math Geol* 36(4):405–424 4, 6, 11
- Caumon G, Tertois AL, Zhang L (2007) Elements for stochastic structural perturbation of stratigraphic models. In: *Proc Petroleum Geostatistics A02, EAGE* 11, 12
- Chambers KT, DeBaun DR, Durlofsky LJ, Taggart IJ, Bernath A, Shen AY, Legarre HA, Goggin DJ (1999) Geologic modeling, upscaling and simulation of faulted reservoirs using complex, faulted stratigraphic grids. In: *Proc SPE Reservoir Simulation Symposium (SPE 51889)* 4
- Charles T, Guéméné J, Corre B, Vincent G, Dubrule O (2001) Experience with the quantification of subsurface uncertainties. In: *Proc SPE Annual Technical Conference and Exhibition (SPE 68703)* 10, 11
- Cherpeau N, Caumon G, Lévy B (2009) Stochastic simulations of structural models. In: *Proc IAMG’09, Stanford, California* 11, 12
- Chilès JP, Aug C, Guillen A, Lees T (2004) Modelling the geometry of geological units and its uncertainty in 3D from structural data: The potential-field method. In: *Proc Orebody Modelling and Strategic Mine Planning* 313–320 6

- Chilès JP, Delfiner P (1999) *Geostatistics: Modeling Spatial Uncertainty*. Series in Probability and Statistics John Wiley and Sons, New York, NY 7
- Cowan EJ, Beatson RK, Ross HJ, Fright WR, McLennan TJ, Evans TR, Carr JC, Lane RG, Bright DV, Gillman AJ, Oshust PA, Titley M (2003) Practical implicit geological modeling. In: Dominy S (ed.) *Proc. 5th International Mining Conference 89–99* Australian Inst. Mining and Metallurgy 6
- Cross TA, Lessenger MA (1999) Construction and application of a stratigraphic inverse model. In: Harbaugh J, Watney W, Rankey E, Slingerland R, Goldstein R, Franseen E (eds.) *Numerical Experiments in Stratigraphy: Recent Advances in Stratigraphic and Sedimentologic Computer Simulations* vol 62:69–83 SEPM Special Publication 8
- Dahlstrom C (1969) Balanced cross section. *Canad J Earth Sci* 6:743,757 8
- Deutsch CV (2002) *Geostatistical Reservoir Modeling*. Oxford University Press, New York, NY 7
- Dunbar J, Cook R (2003) Palinspastic reconstruction of structure maps: an automated finite element approach with heterogeneous strain. *J Struct Geol* 26:1021–1036 8
- Durand-Riard P, Caumon G, Muron P (2010) Balanced restoration of geological volumes with relaxed meshing constraints. *Comp Geosci* 36(4):441–452 8
- Dynamic Graphics (2009) EarthVision software URL <http://www.dgi.com> 2
- Elliott D (1983) The construction of balanced cross-sections. *J Struct Geol* 5(2):101 8
- Euler N, Sword CH, Dulac JC (1998) A new tool to seal a 3D earth model: a cut with constraints. In: *Proc 68th SEG Annual Meeting* 710–713 6, 11
- Evensen G (2007) *Data assimilation: The ensemble Kalman filter*. Springer Verlag, Berlin 13
- Fisher TR, Wales RQ (1992) Rational splines and multidimensional geologic modeling. In: *Computer Graphics in Geology* vol 41:17–28 Springer, Berlin 3

- Frank T, Tertois AL, Mallet JL (2007) 3D-reconstruction of complex geological interfaces from irregularly distributed and noisy point data. *Comp Geosci* 33:932–943 6, 7
- Fremming N (2002) 3D geological model construction using a 3D grid. In: *Proc. ECMOR VIII European Conference on Mathematics of Oil Recovery*. 4
- Frøyland LA, Lakså A, Strom K, Pajchel J (1993) A 3D cellular, smooth boundary representation modelling system for geological structures. In: *EAGE 55th Meeting and technical Exhibition* 6
- Gemcom (2009) Gems and Surpack software URL <http://www.gemcomsoftware.com> 2
- Gjøystdal H, Reinhardsen JE, Åstebøl K (1985) Computer representation of complex three-dimensional geological structures using a new solid modeling technique. *Geophys Prospect* 33(8):1195–1211 3, 4, 6
- Goff JA (2000) Simulation of stratigraphic architecture from statistical and geometrical characterizations. *Math Geol* 32(7):765–786 11
- Goovaerts P (1997) *Geostatistics for natural resources evaluation*. Applied Geostatistics. Oxford University Press, New York, NY 7
- Gratier J, Guillier B, Delorme A (1991) Restoration and balance of a folded and faulted surface by best-fitting of finite elements: principles and applications. *J Struct Geol* 13(1):111–1115 8
- Gringarten E, Arpat G, Haouesse M, Dutranois A, Deny L, Jayr S, Tertois A, Mallet J, Bernal A, Nghiem L (2008) New Grids for Robust Reservoir Modeling. In: *SPE Annual Technical Conference and Exhibition (SPE 116649)* 6, 7
- Guillen A, Calcagno P, Courrioux G, Joly A, Ledru P (2008) Geological modelling from field data and geological knowledge part II: modelling validation using gravity and magnetic data inversion. *Phys Earth Planet Inter* 171(1-4):158–169 11
- Guiziou JL, Mallet JL, Madriaga R (1996) 3-d seismic reflection tomography on top of the GOCAD depth modeler. *Geophys* 61(5):1499–1510 6
- Guzofski CA, Mueller JP, Shaw JH, Muron P, Medwedeff DA, Bilotti F, Rivero C (2009) Insights into the mechanisms of fault-related folding provided by volumetric structural restorations using spatially varying mechanical constraints. *AAPG Bull* 93(4):479–502 8, 9

- Harbaugh J, Watney W, Rankey E, Slingerland R, Goldstein R, Franseen E (eds.) (1999) Numerical Experiments in Stratigraphy: Recent Advances. In Stratigraphic and Sedimentologic Computer Simulations vol 62. SEPM Spec Pub 8
- Hoaglund J, Pollard D (2005) Dip and anisotropy effects on flow using a vertically skewed model grid. *Ground Water* 41(6):841–846 4
- Hoffman KS, Neave JW, Klein RT (2003) Streamlining the workflow from structure model to reservoir grid. In: SPE Annual Conference and Technical Exhibition (SPE 66384) 3, 4
- Hoffmann CM (1989) Geometric and Solid Modeling: An Introduction. Morgan Kaufmann, San Francisco 338 p. 4
- Holden L, Mostad P, Nielsen B, Gjerde J, Townsend C, Ottesen S (2003) Stochastic structural modeling. *Math Geol* 35(8):899–914 11
- Hollund K, Mostad PF, Nielsen BF, Holden L, Gjerde J, Contursi MG, McCann AJ, Townsend C, Sverdrup E (2002) Havana: a fault modeling tool. In: Hydrocarbon Seal Quantification, Norwegian Petroleum Society Conference vol 11 of NPF Spec Pub, Elsevier 11
- Houlding S (1994) 3D geoscience modeling, computer techniques for geological characterization. Springer, Berlin 6
- Hu L, Blanc G, Noetinger B (2001) Gradual deformation and iterative calibration of sequential stochastic simulations. *Math Geol* 33(4):475–489 13
- Intrepid Geophysics (2009) 3D Geomodeller software URL <http://www.intrepid-geophysics.com> 2
- Jayr S, Gringarten E, Tertois AL, Mallet J, Dulac JC (2009) The need for a correct geological modelling support: the advent of the uvt-transform. *First Break* 26(10):73–79 6, 7
- Jessell M (2001) Three-dimensional geological modelling of potential-field data. *Comp Geosci* 27(4):455–465 4
- Jessell M, Valenta R (1996) Structural geophysics: Integrated structural and geophysical modeling. In: de Paor DG (ed.) Structural geology and personal computers 303–323, Elsevier 8
- JOA (2009) JOA Jewel Suite software URL <http://www.jewelsuite.com> 2

- Johnson C, Jones T (1988) Putting geology into reservoir simulations: a three-dimensional modeling approach. In: SPE Annual Conference and Technical Exhibition (SPE 18321) 4
- Journel AG (1993) Resampling from stochastic simulations. *Environ Ecol Stat* 1:63–83 10
- Journel AG, Huijbregts C (1978) *Mining Geostatistics*. Academic Press, NY; Blackburn Press, NJ (reprint, 2004) 7
- Kaven J, Mazzeo R, Pollard D (2009) Constraining surface interpolations using elastic plate bending solutions with applications to geologic folding. *Math Geosci* 41(1):1–14 8
- Kedzierski P, Caumon G, Mallet JL, Royer JJ, Durand-Riard P (2008) 3D marine sedimentary reservoir stochastic simulation accounting for high resolution sequence stratigraphy and sedimentological rules. In: Ortiz J, Emery X (eds.) *Proc. eighth Geostatistical Geostatistics Congress vol 2* 657–666. Gecamin ltd, Santiago 7
- de Kemp EA (1999) Visualization of complex geological structures using 3-D Bézier construction tools. *Comp and Geosci* 25(5):581–597 3
- de Kemp EA, Sprague KB (2003) Interpretive tools for 3D structural geological modeling part I: Bézier-based curves, ribbons and grip frames. *GeoInformatica* 7(1):55–71 3
- Lasseter T, Jackson S (2004) Improving integrated interpretation accuracy and efficiency using a single consistent reservoir model from seismic to simulation. *Lead Edge* 23(11):1118–1121 4
- Lecour M, Cognot R, Duvinage I, Thore P, Dulac JC (2001) Modeling of stochastic faults and fault networks in a structural uncertainty study. *Petrol Geosci* 7:S31–S42 11
- Ledez D (2002) Euclidean distance mapping: Geological applications in: *Terra Nostra*, (Proc IAMG) vol 04:25–30, Berlin 7
- Lelièvre P, Oldenburg D, Williams N (2008) Constraining geophysical inversions with geologic information. In: *SEG Technical Program Expanded Abstracts* 27(1):1223–1227 13
- Lemon AM, Jones NL (2003) Building solid models from boreholes and user-defined cross-sections. *Comp Geosci* 29:547–555 4, 6

- Maerten F, Pollard DD, Karpuz R (2000) How to constrain 3-D fault continuity and linkage using reflection seismic data: A geomechanical approach. AAPG Bull 84(9):1311–1324 6
- Maerten L, Maerten F (2006) Chronologic modeling of faulted and fractured reservoirs using geomechanically based restoration: Technique and industry applications. AAPG Bull 90(8):1201 8
- Maharaja A, Journel AG, Caumon G, Strebelle S (2008) Assessment of net-to-gross uncertainty at reservoir appraisal stage: Application to a turbidite reservoir offshore west africa. In: Ortiz J, Emery X (eds.) Proc eighth Geostatistics Congress vol 2:707–716 Gecamin ltd, Santiago 10
- Mallet JL (1988) Three dimensional graphic display of disconnected bodies. Math Geol 122:977–990 3, 4
- Mallet JL (2002) Geomodeling. Applied Geostatistics, Oxford University Press, New York, NY 4, 8
- Mallet JL (2004) Space-time mathematical framework for sedimentary geology. Math Geol 36(1):1–32 6, 7
- Mann CJ (1993) Uncertainty in geology in: Davis JC, Herzfeld UC (eds.) Computers in Geology – 25 years of progress number 5. In IAMG studies in Mathematical Geology chap 20:241–254. Oxford University Press, New York, NY 2
- Manzocchi T, Carter J, Skorstad A, Fjellvoll B, Stephen K, Howell JA, Matthews JD, Walsh JJ, Nepveu M, Bos C, et al. (2008) Sensitivity of the impact of geological uncertainty on production from faulted and unfaulted shallow-marine oil reservoirs: objectives and methods. Petrol Geosci 14(1):3–15 10
- Massonnat G (1999) Breaking of a paradigm: Geology can provide 3D complex probability fields for stochastic facies modeling. In: Annual Technical Conference and Exhibition (SPE 56652) 7
- Matheron G (1970) La théorie des variables régionalisées et ses applications: Fascicule 5, Centre de Géostatistique, Ecole des Mines de Paris, Fontainebleau 7
- Mello UT, Henderson ME (1997) Techniques for including large deformations associated with salt and fault motion in basin modeling. Mar Petrol Geol 14(5):551–564 6, 8

- Midland Valley (2009) 4D Move software URL <http://www.mve.com> 2
- Moretti I (2008) Working in complex areas: New restoration workflow based on quality control, 3D and 3D restorations. *Mar Petrol Geol* 25(3):205–218 8
- Moyen R, Mallet JL, Frank T, Leflon B, Royer JJ (2004) 3D-Parameterization of the 3D geological space - The GeoChron model. In: *Proc European Conference on the Mathematics of Oil Recovery (ECMOR IX)* 7
- Muron P (2005) Méthodes numériques 3D de restauration des structures géologiques faillées. PhD dissertation, INPL, Nancy, France 8
- Mustapha H, Mustapha K (2007) A new approach to simulating flow in discrete fracture networks with an optimized mesh. *SIAM J Sci Comp* 29:1439–1459 6
- Norris R, Massonnat G, Alabert F (1993) Early quantification of uncertainty in the estimation of oil-in-place in a turbidite reservoir. In: *SPE Annual Technical Conference and Exhibition (SPE 26490)* 10
- Owen S (1998) A survey of unstructured mesh generation technology. In: *Proc 7th International Meshing RoundTable* 239–267 6
- Paradigm Geophysical (2009) Gocad and SKUA Suite software URL <http://www.pdgm.com> 2, 3
- Piegl LA, Tiller W (1997) *The NURBS Book*. Monographs in Visual Communications, 2nd edn, Springer, Berlin 3
- Rémy N, Boucher A, Wu J (2008) *Applied Geostatistics with SGeMS - A User's guide*. Cambridge University Press, New York, NY 7
- Rouby D, Xiao H, Suppe J (2000) 3-D restoration of complexly folded and faulted surfaces using multiple unfolding mechanisms. *AAPG Bull* 84(6):805–829 8
- Roxar (2009) Irap rms software URL <http://www.roxar.com> 2, 3
- Sambridge M (1999a) Geophysical inversion with a neighbourhood algorithm-I: searching a parameter space. *Geophys J Intern* 138:479–494 13
- Sambridge M (1999b) Geophysical inversion with a neighbourhood algorithm-II: appraising the ensemble. *Geophys J Intern* 138(3):727–746 13

- Scheidt C, Caers JK (2009) Representing spatial uncertainty using distances and kernels. *Math Geosci* 41(4):397–419 13
- Schlumberger (2009) Petrel software URL <http://www.slb.com/content/services/software> 2, 3
- Sprague KB, de Kemp EA (2005) Interpretive tools for 3-D structural geological modelling part II: Surface design from sparse spatial data. *GeoInformatica* 9(1):5–32 3
- Suzuki S, Caers J (2008) A distance-based prior model parameterization for constraining solutions of spatial inverse problems. *Math Geosci* 40(4):445–469 13
- Suzuki S, Caumon G, Caers J (2008) Dynamic data integration for structural modeling: model screening approach using a distance-based model parameterization. *Comp Geosci* 12(1):105–119 13, 14
- Swanson D (1988) A new geological volume computer modeling system for reservoir description. In: *SPE Annual Conference and Technical Exhibition (SPE 17579)* 4
- Sword CH (1991) Building flexible interactive, geologic models. In: *Proc 61st SEG Annual International Meeting* 1465–1467 6
- Tarantola A (1987) *Inverse Problem Theory*. Elsevier, New York 12
- Tarantola A (2006) Popper, Bayes and the inverse problem. *Nat Phys* 2(8):492–494 15
- Tertois AL, Mallet JL (2007) Editing faults within tetrahedral volumes in real time. In: Jolley S, Barr D, Walsh J, Knipe R (eds.) *Structurally Complex Reservoirs Spec Pub vol 292:89–101*, Geological Society, London 11
- Thibault M, Gratier JP, Léger M, Morvan JM (1996) An inverse method for determining three dimensional fault with thread criterion : strike slip and thrust faults. *J Struct Geol* 18:1127–1138 8
- Thibert B, Gratier JP, Morvan JM (2005) A direct method for modeling and unfolding developable surfaces and its application to the Ventura Basin (California). *J Struct Geol* 27(2):303–316 8
- Thomas AL (1993) *Poly3D: A Three-Dimensional, Polygonal Element, Displacement Discontinuity Boundary Element Computer Program with Applications to Fractures, Faults, and Cavities in the Earth's Crust*. Master's dissertation, Stanford University 6

- Thore P, Shtuka A, Lecour M, Ait-Ettajer T, Cognot R (2002) Structural uncertainties: determination, management and applications. *Geophys* 67(3):840–852 10, 12
- Tournois J, Wormser C, Alliez P, Desbrun M (2009) Interleaving Delaunay refinement and optimization for practical isotropic tetrahedron mesh generation. *ACM Trans Graph* 28(3):75–6
- Turner AK (ed.) (1992) *Three-Dimensional Modeling with Geoscientific Information Systems*. Vol 354 of NATO-ASI, Math. and Phys. Sciences, Kluwer Academic Publishers, Dordrecht 3
- Vail PR (1987) Seismic stratigraphy interpretation using sequence stratigraphy, part 1: seismic stratigraphy interpretation procedure. In: Bally AW (ed.) *Atlas of Seismic Stratigraphy* 27(1):1–10, AAPG Studies in Geology 7
- Wheeler HF (1958) Time-stratigraphy. *Bull AAPG* 42(5):1047–1063 7
- Wu XH, Parashkevov RR (2009) Effect of grid deviation on flow solutions. *SPE J* 14(1):67–77 6
- Zanchi A, de Donatis M, Gibbs A, Mallet JL (2009) Imaging geology in 3D. *Comp Geosci* 35(1) 3
- Zaparo (2009) Leapfrog software URL <http://www.leapfrog3d.com> 2
- Zhang H, Thurber C (2005) Adaptive mesh seismic tomography based on tetrahedral and Voronoi diagrams: application to Parkfield, California. *J Geophys Res* 110(B04303) 6
- Zhong D, Li M, Song L, Wang G (2006) Enhanced NURBS modeling and visualization for large 3D geoengineering applications: An example from the Jinping first-level hydropower engineering project, China. *Comp Geosci* 32(9):1270–1282 3, 6